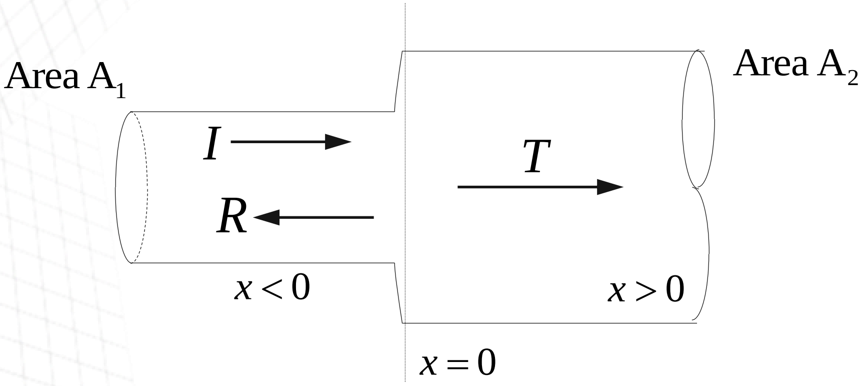
❖ Duct Acoustics

● Plane wave

- A sound propagation in pipes with different cross-sectional area
- If the wavelength of sound is large in comparison with the diameter of the pipe the sound propagates as an one-dimensional wave ($\lambda >> d$ → 1-d wave)



$$p' = Ie^{i\omega(t - x/c)} + R\,e^{i\omega(t + x/c)} \quad \text{in } x < 0$$

$$= Te^{i\omega(t - x/c)} \quad \text{in } x > 0$$

I : 입사, R : 반사, T : 투과

❖ Duct Acoustics

- The mass flux into the junction must equal the mass flux out

$$\rho_0 A_1 u_1 = \rho_0 A_2 u_2$$

- The velocity must equal at both sides of the junction

$$\frac{A_1}{\rho_0 c}\left(I - R\right) = \frac{A_2}{\rho_0 c} T$$

- Energy flux$)_{in}$ = Energy flux$)_{out}$

$$A_1 p_1^{'} u_1 = A_2 p_2^{'} u_2$$

- The pressure of both sides of junction is continuous

$$I + R = T$$

❖ Duct Acoustics

- The amplitudes of other wave, R and T ,are can be solve from above the relations

$$R = \frac{A_1 - A_2}{A_1 + A_2} I, \qquad T = \frac{2A_1}{A_1 + A_2} I$$

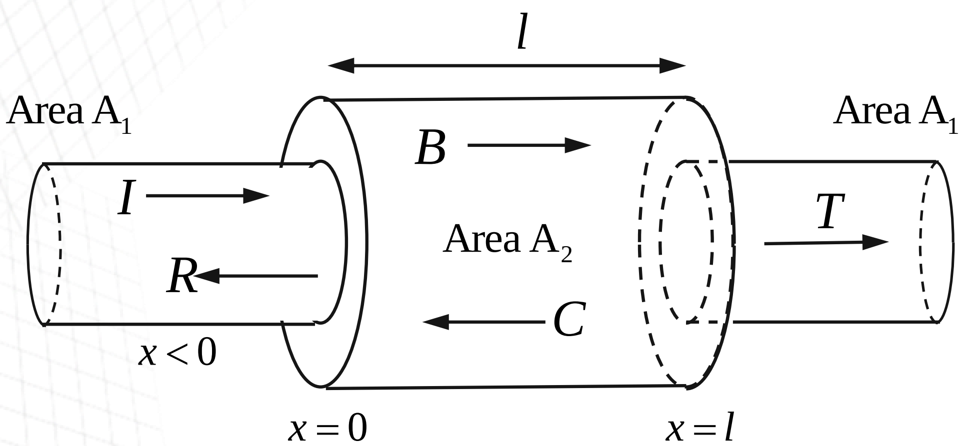
- The transmission loss , L_T is symmetric in A_1 and A_2

$$L_T = 10\log_{10}\left(\frac{\text{incident power}}{\text{transmitted power}}\right)$$

$$= 10\log_{10}\left(\frac{A_1 I^2}{A_2 T^2}\right) = 10\log_{10}\left(\frac{(A_1 + A_2)^2}{4A_1A_2}\right)$$

Duct Acoustics

- A single expansion-chamber 'silencer'
  - The simple muffler that is a used in car 'silencer' consists of inlet and outlet pipes with cross-sectional area A_1, and expansion chamber between them of cross-sectional area A_2 and length l



❖ Duct Acoustics

- The first area change occurs at *x=0* and the second occurs at *x=l*.

$$
\begin{aligned}
p' &= Ie^{i\omega(t-x/c)} + R\,e^{i\omega(t+x/c)} && \text{in } x<0 \\
&= Be^{i\omega(t-x/c)} + Ce^{i\omega(t+x/c)} && \text{in } 0<x<l \\
&= Te^{i\omega(t-x/c)} && \text{in } l<x
\end{aligned}
$$

- The condition of continuity of mass flux,

$$
\begin{aligned}
A_1(I-R) &= A_2(B-C) && \text{at } x=0 \\
A_1Te^{-i\omega l/c} &= A_2\left(Be^{-i\omega l/c} - Ce^{-i\omega l/c}\right) && \text{at } x=l
\end{aligned}
$$

- The condition of continuity of pressure

$$
\begin{aligned}
I + \mathrm{R} &= \mathrm{B} + \mathrm{C} && \text{at } x=0 \\
Te^{-i\omega l/c} &= Be^{-i\omega l/c} + Ce^{-i\omega l/c} && \text{at } x=l
\end{aligned}
$$

❖ Duct Acoustics

- The algebraic equation when solved for *R* and *T*

$$R = \frac{\left(\frac{A_1}{A_2} - \frac{A_2}{A_1}\right) Ii \sin\frac{\omega l}{c}}{2\cos\omega l/c + i\left(\frac{A_1}{A_2} + \frac{A_2}{A_1}\right)\sin\frac{\omega l}{c}} \qquad T = \frac{2Ie^{i\omega l/c}}{2\cos\omega l/c + i\left(\frac{A_1}{A_2} + \frac{A_2}{A_1}\right)\sin\frac{\omega l}{c}}$$

- However, the simple **'silencer'** does not reduce the total energy of sound in the system.

$$|R|^2 + |T|^2 = |I|^2$$

- Reducing the acoustic energy of transmitted wave

→ Increasing in the reflected wave

- Sound absorbing material

→ reduce the acoustic energy by converting it into heat or vibration

Duct Acoustics

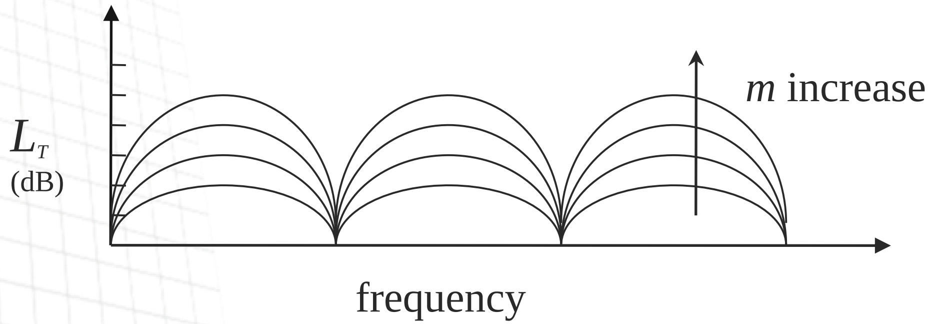
- The transmission loss , L_T is $L_T = 10\log_{10}\left(\frac{|I|^2}{|T|^2}\right)$

$$L_T = 10\log_{10}\left[1+\frac{1}{4}\left(\frac{A_1}{A_2}-\frac{A_2}{A_1}\right)^2 \sin\left(\frac{\omega l}{c}\right)\right]$$

- The transmission loss is maximum at frequencies for which

$$\sin \omega l / c = \pm 1 \qquad \text{i.e.} \quad \omega = \frac{\pi c}{2l}, \frac{3\pi c}{2l}, \frac{5\pi c}{2l} \ldots \text{etc}$$

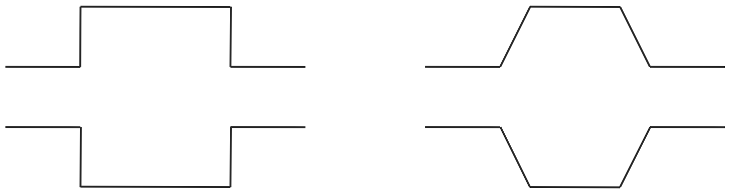
- The effect of expansion ratio $m = A_2 / A_1$



Duct Acoustics

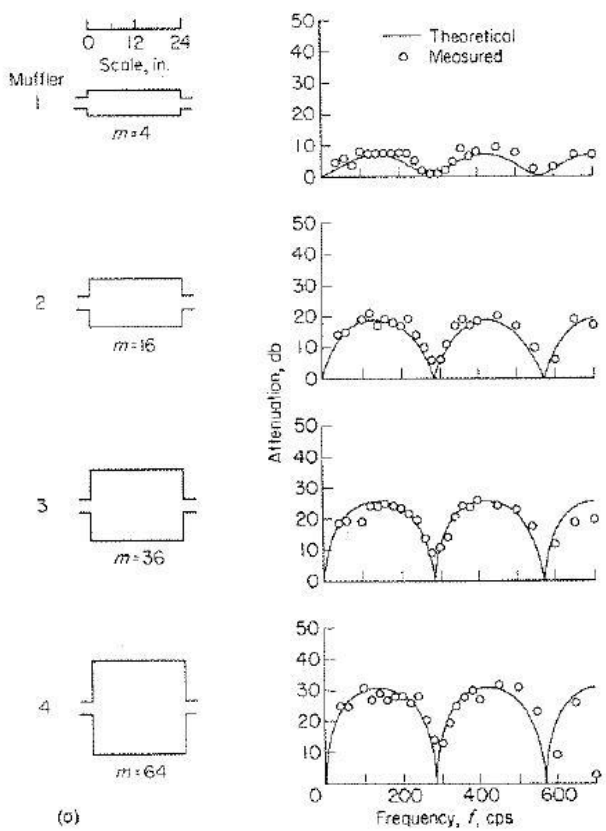
Note

- 'Tuning' for dominant frequencies of noise
- Theory work for only $\lambda \gg d$ "Low frequency wave only"
- High frequency waves behave like 3-D
- Also, the geometrical shape of the duct is not important (provided the area change occurs in a distance short in comparison with the wavelength

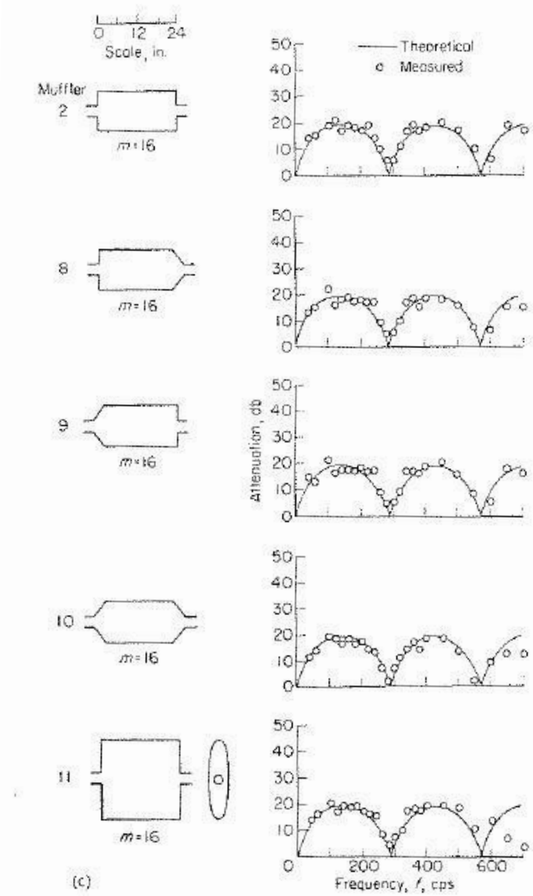


❖Duct Acoustics

Ref. **“Theoretical and experimental investigation of mufflers with comments on engine-exhaust muffler design”, Davis et al. NACA 1192(1954)**

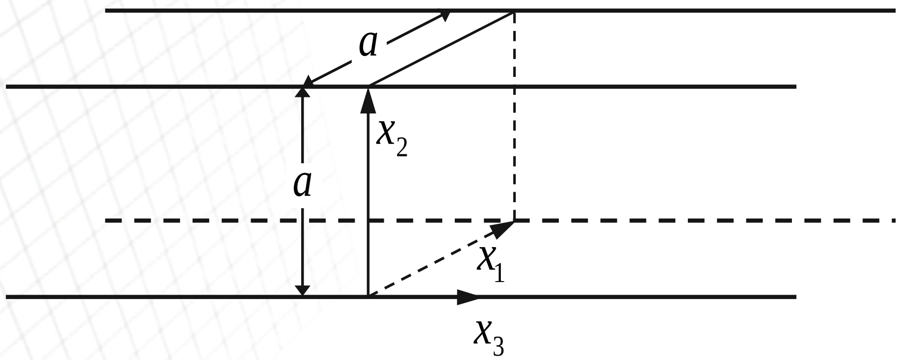


Effect of expansion chamber ratio



Effect of expansion chamber shape

❖ Higher order modes



- As an illustration, the sound of frequency ω in a rigid walled duct of square cross-section with sides of length a is considered

$$p'(\mathbf{x},t) = f(x_1)g(x_2)h(x_3)e^{i\omega t}$$

- With substitution for p' into the wave equation,

$$\frac{f''}{f} = -\frac{g''}{g} - \frac{h''}{h} - \frac{\omega^2}{c^2} = -\alpha^2$$

Higher order modes

- Since a wall boundary condition is applied, function *f* is derived like this

$$f(x_1) = A_1 \cos\left(\frac{m\pi x_1}{a}\right) \quad , \quad \text{for some integer } m$$

- Similarly function *g* is derived like this

$$g(x_2) = A_2 \cos\left(\frac{n\pi x_1}{a}\right) \quad , \quad \text{for some integer } n$$

- Finally, function *h* is derived to the propagation form

$$h(x_3) = A_{mn} e^{-ik_{mn} x_3} + B_{mn} e^{ik_{mn} x_3} \qquad k_{mn} = \sqrt{\frac{\omega^2}{c^2} - \frac{\pi^2}{a^2}\left(m^2 + n^2\right)}$$

- The axial phase speed, $c_p = \omega / k_{mn}$ is now a function of the mode number and the propagation of a group of waves will cause them to disperse.

Higher order modes

- The pressure perturbation in the *(m,n)* mode has the form

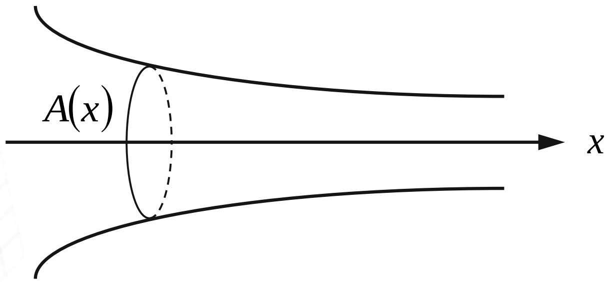
$$p'(\mathbf{x},t) = \cos\left(\frac{m\pi x_1}{a}\right)\cos\left(\frac{n\pi x_2}{a}\right)\left[A_{mn}e^{-ik_{mn}x_3} + B_{mn}e^{ik_{mn}x_3}\right]e^{i\omega t}$$

- When k_{mn} is real, the pressure perturbation equation represents that waves are propagating down the x_3 axis with phase speed.

- When k_{mn} is purely imaginary, i.e. exceeds the cut-off frequency, the strength of mode varies exponentially with distance along the pipe. Such disturbances are evanescent

Pipes of varying cross-section

- Wave equation



  - If the pipe diameter is small in comparison with both the acoustic wavelength and the length scale over which the cross-sectional area change, most particle motions are longitudinal.
  - Conservation of mass $A\dfrac{\partial \rho}{\partial t} = -\rho_0 \dfrac{\partial}{\partial x}(uA)$
  - Linearized momentum equation is $\rho_0 \dfrac{\partial u}{\partial t} = -\dfrac{\partial p'}{\partial x}$
  - Modified wave equation $\dfrac{A}{c^2}\dfrac{\partial^2 p'}{\partial t^2} = \dfrac{\partial}{\partial x}\left(A\dfrac{\partial p'}{\partial x}\right)$

Pipes of varying cross-section

Application to the 'exponential horn'

- Evaluation of the case of 'exponential horn' which cross-sectional area defined as, $A(x)=A_0 e^{\alpha x}$

- For such an area variation of wave equation simplifies to

$$\frac{1}{c^2}\frac{\partial^2 p'}{\partial t^2}=\frac{\partial^2 p'}{\partial x^2}+\alpha\frac{\partial p'}{\partial x}$$

- The pressure perturbation in sound waves of frequency ω then has the form

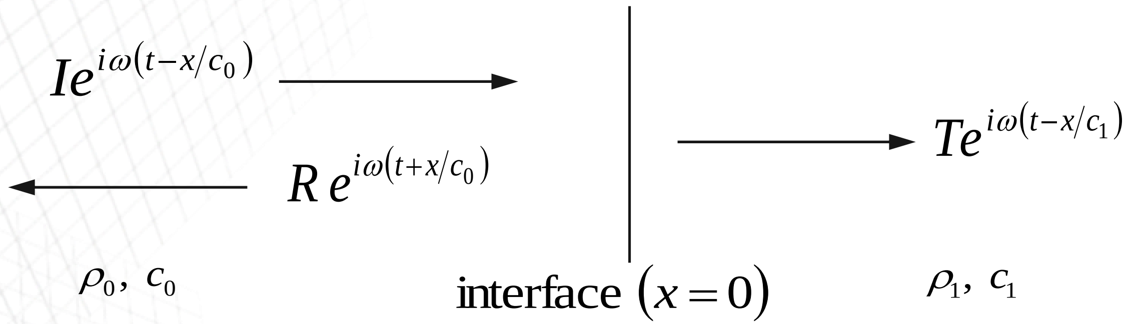
$$p'(x,t)=e^{-\alpha x/2}\left\{Ae^{i(\omega t-kx)}+Be^{i(\omega t+kx)}\right\}$$

- Disturbance with $\omega > \alpha c/2$ propagates and the pressure but not the energy flux attenuates during propagation, while lower frequency modes are 'cut-off'

❖ Normal transmission

● Physics at the interface

- When a sound wave crosses an interface between two different fluids some of the acoustic energy is usually reflected.



- There are two boundary conditions
  - The pressure on the two sides of the boundary must be equal
  - The particle velocities normal to the interface must be equal

$$\lambda = cT = \frac{c}{f} = \frac{2\pi c}{\varpi} \qquad \lambda_0 = \frac{2\pi c_0}{\varpi} \qquad \lambda_1 = \frac{2\pi c_1}{\varpi}$$

❖ Normal transmission

- The pressure must be equal at the interface : $I+R=T$
- The particle velocities normal to the interface must be equal

$$\frac{I}{\rho_0 c_0} - \frac{R}{\rho_0 c_0} = \frac{T}{\rho_1 c_1}$$

- The result pressure coefficients, R and T , are determined with I

$$R = \left\{ \frac{\rho_1 c_1 - \rho_0 c_0}{\rho_1 c_1 + \rho_0 c_0} \right\} I \qquad T = \left\{ \frac{2\rho_1 c_1}{\rho_1 c_1 + \rho_0 c_0} \right\} I$$

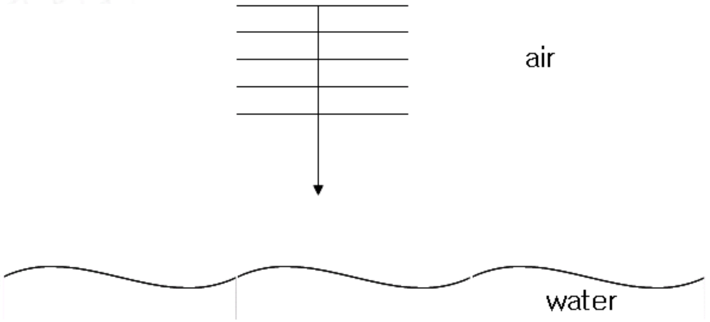
- Velocity Transmission Coefficient : $\dfrac{T / \rho_1 c_1}{I / \rho_0 c_0} = \dfrac{2\rho_0 c_0}{\rho_1 c_1 + \rho_0 c_0}$

- The energy flux of the incident wave per unit cross sectional area is equal to that of the reflected and transmitted waves

$$\frac{R^2}{\rho_0 c_0} + \frac{T^2}{\rho_1 c_1} = \frac{(\rho_1 c_1 - \rho_0 c_0)^2 I^2}{(\rho_1 c_1 + \rho_0 c_0)^2 \rho_0 c_0} + \frac{4\rho_1^{\ 2} c_1^{\ 2} I^2}{(\rho_1 c_1 + \rho_0 c_0)^2 \rho_1 c_1} = \frac{I^2}{\rho_0 c_0}$$

❖ Normal transmission

- Reflection from a high and low impedance fluid



- A typical example is aerial sound waves incident onto a water surface. ($\rho_0 c_0 \ll \rho_1 c_1$)

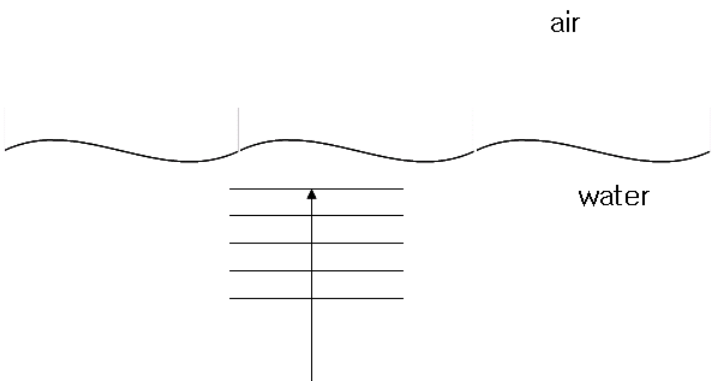
$$R = I \,, \qquad T = 2I$$

- Velocity transmission coefficient $= \dfrac{2\rho_0 c_0}{\rho_1 c_1 + \rho_0 c_0} \approx 0$

so, the transmission wave carries negligible energy

❖ Normal transmission

- Reflection from a high and low impedance fluid



- In the opposite case, for sound in water incident onto a free surface with air, the reflected and transmitted waves are

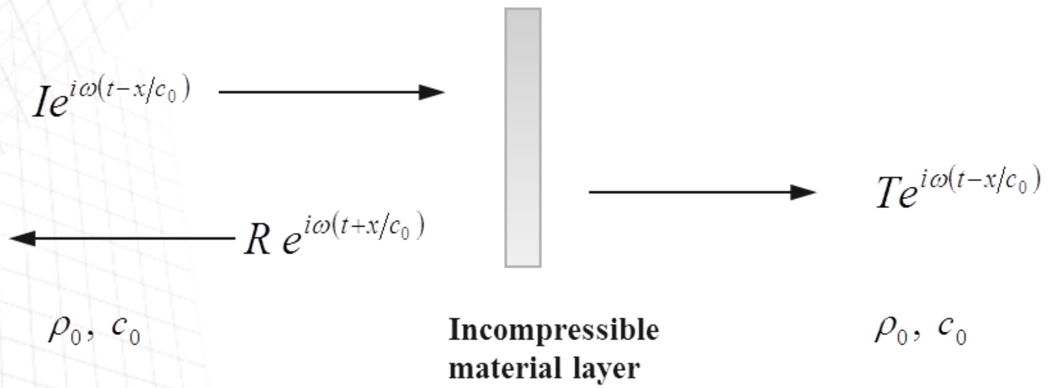
$$R = -I\,, \qquad T = 0$$

- The acoustic energy is totally reflected

Sound propagation through walls

Effect of a wall in transmission

- A sound wave normally incident on a plane material layer partitioning a fluid which has uniform acoustic properties, $\rho_0 c_0$
- Some sound will be reflected from the layer and some will be transmitted through the wall



Sound propagation through walls

- There are two boundary conditions that must be satisfied at all times and points
  - The velocity of the wall must be equal to wave of each side
  - A pressure difference across the wall in order to provide the force necessary to accelerate unit area of the surface of material
- By continuity the velocity of wall,

$$u = (I - R)\frac{e^{i\omega t}}{\rho_0 c_0} = \frac{T}{\rho_0 c_0} e^{i\omega t}$$

- The pressure difference is the net force of mass per unit area of the wall

$$p_1^{'} = (I + R)e^{i\omega t}$$
$$p_2^{'} = Te^{i\omega t}$$

$$(I + R - T)e^{i\omega t} = m\frac{\partial u}{\partial t} = m\frac{i\omega T}{\rho_0 c_0} e^{i\omega t}$$

❖ Sound propagation through walls

- The result pressure coefficients, R and T, are determined with I

$$R = \left\{ \frac{i\omega m}{2\rho_0 c_0 + i\omega m} \right\} I \qquad T = \left\{ \frac{2\rho_0 c_0}{2\rho_0 c_0 + i\omega m} \right\} I$$

- Surface Impedance

$$\frac{p_1^{'}}{u} = \rho_0 c_0 \frac{(I+R)}{(I-R)} = \rho_0 c_0 \frac{1+i\omega m}{1-i\omega m}$$

$$\frac{p_2^{'}}{u} = \rho_0 c_0 \frac{(I+R)}{T} = \rho_0 c_0 + i\omega m$$

- Energy transmitted

$$\frac{|T|^2}{\rho_0 c_0} = \frac{4\rho_0^2 c_0^2}{4\rho_0^2 c_0^2 + \omega^2 m^2} \frac{I^2}{\rho_0 c_0}$$

Sound propagation through walls

- The transmission loss is dependent on the frequency ω.

$$L_T = 10\log_{10}\left(\frac{4\rho_0^2 c_0^2}{4\rho_0^2 c_0^2 + \omega^2 m^2}\right)^{-1}$$

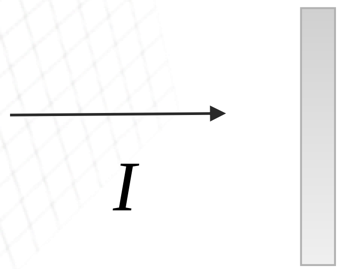
- For high frequency($\omega m \gg \rho_0 c_0$), the sound waves mostly reflected

- For low frequency($\omega m \ll \rho_0 c_0$), the sound waves mostly travels through the wall with very little attenuation

- "Low frequency waves get through a massive wall easily, while high frequency waves are effectively stopped"

❖ Sound propagation through walls

- Example) Attenuation by a wall



$$m = 50 \, {}^{kg}\!/_{m^2}$$

$$\rho_0 c_0 = 410 \, {}^{kg}\!/_{m^2 \sec}$$

- Transmission loss $\quad L_T = 10 \log_{10}\left(\frac{|I|^2}{|T|^2}\right) \quad L_T = 10 \log_{10}\left(\frac{4\rho_0^2 c_0^2}{4\rho_0^2 c_0^2 + \omega^2 m^2}\right)^{-1}$

$$for \; f = 10\,Hz \qquad L_T \approx 12dB$$

$$for \; f = 1kHz \qquad L_T \approx 50dB$$